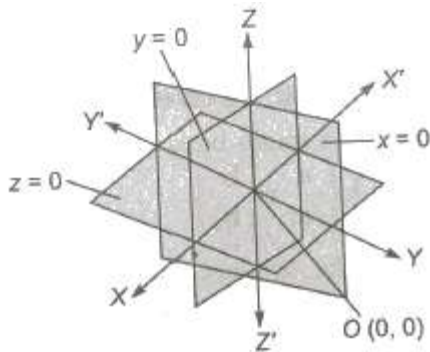


# Mathematics Notes for Class 12 chapter 11.

## Three Dimensional Geometry

### Coordinate System

The three mutually perpendicular lines in a space which divides the space into eight parts and if these perpendicular lines are the coordinate axes, then it is said to be a coordinate system.



### Sign Convention

Octant Coordinate	x	y	z
OXYZ	+	+	+
OX'YZ	-	+	+
OXY'Z	+	-	+
OXYZ'	+	+	-
OX'Y'Z	-	-	+
OX'YZ'	-	+	-
OXY'Z'	+	-	-
OX'Y'Z'	-	-	-

### Distance between Two Points

Let  $P(x_1, y_1, z_1)$  and  $Q(x_2, y_2, z_2)$  be two given points. The distance between these points is given by

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

The distance of a point  $P(x, y, z)$  from origin  $O$  is

$$OP = \sqrt{x^2 + y^2 + z^2}$$

### Section Formulae

(i) The coordinates of any point, which divides the join of points  $P(x_1, y_1, z_1)$  and  $Q(x_2, y_2, z_2)$  in the ratio  $m : n$  internally are

$$(mx_2 + nx_1 / m + n, my_2 + ny_1 / m + n, mz_2 + nz_1 / m + n)$$

(ii) The coordinates of any point, which divides the join of points  $P(x_1, y_1, z_1)$  and  $Q(x_2, y_2, z_2)$  in the ratio  $m : n$  externally are

$$(mx_2 - nx_1 / m - n, my_2 - ny_1 / m - n, mz_2 - nz_1 / m - n)$$

(iii) The coordinates of mid-point of  $P$  and  $Q$  are

$$(x_1 + x_2 / 2, y_1 + y_2 / 2, z_1 + z_2 / 2)$$

(iv) Coordinates of the centroid of a triangle formed with vertices  $P(x_1, y_1, z_1)$  and  $Q(x_2, y_2, z_2)$  and  $R(x_3, y_3, z_3)$  are

$$(x_1 + x_2 + x_3 / 3, y_1 + y_2 + y_3 / 3, z_1 + z_2 + z_3 / 3)$$

(v) **Centroid of a Tetrahedron**

If  $(x_1, y_1, z_1)$ ,  $(x_2, y_2, z_2)$ ,  $(x_3, y_3, z_3)$  and  $(x_4, y_4, z_4)$  are the vertices of a tetrahedron, then its centroid  $G$  is given by

$$(x_1 + x_2 + x_3 + x_4 / 4, y_1 + y_2 + y_3 + y_4 / 4, z_1 + z_2 + z_3 + z_4 / 4)$$

### Direction Cosines

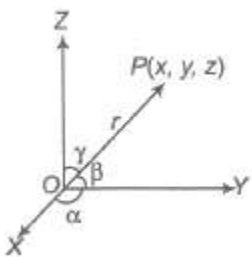
If a directed line segment  $OP$  makes angle  $\alpha$ ,  $\beta$  and  $\gamma$  with  $OX$ ,  $OY$  and  $OZ$  respectively, then  $\cos \alpha$ ,  $\cos \beta$  and  $\cos \gamma$  are called direction cosines of  $OP$  and it is represented by  $l$ ,  $m$ ,  $n$ .

i.e.,

$$l = \cos \alpha$$

$$m = \cos \beta$$

$$\text{and } n = \cos \gamma$$



If  $OP = r$ , then coordinates of  $OP$  are  $(lr, mr, nr)$

(i) If  $l, m, n$  are direction cosines of a vector  $r$ , then

(a)  $r = |r|(li + mj + nk) \Rightarrow r = li + mj + nk$

(b)  $l^2 + m^2 + n^2 = 1$

(c) Projections of  $r$  on the coordinate axes are

(d)  $|r| = l|r|, m|r|, n|r| / \sqrt{\text{sum of the squares of projections of } r \text{ on the coordinate axes}}$

(ii) If  $P(x_1, y_1, z_1)$  and  $Q(x_2, y_2, z_2)$  are two points, such that the direction cosines of  $PQ$  are  $l, m, n$ . Then,

$$x_2 - x_1 = l|PQ|, y_2 - y_1 = m|PQ|, z_2 - z_1 = n|PQ|$$

These are projections of  $PQ$  on  $X, Y$  and  $Z$  axes, respectively.

(iii) If  $l, m, n$  are direction cosines of a vector  $r$  and  $a, b, c$  are three numbers, such that  $l/a = m/b = n/c$ .

Then, we say that the direction ratio of  $r$  are proportional to  $a, b, c$ .

Also, we have

$$l = a / \sqrt{a^2 + b^2 + c^2}, m = b / \sqrt{a^2 + b^2 + c^2}, n = c / \sqrt{a^2 + b^2 + c^2}$$

(iv) If  $\theta$  is the angle between two lines having direction cosines  $l_1, m_1, n_1$  and  $l_2, m_2, n_2$ , then

$$\cos \theta = l_1 l_2 + m_1 m_2 + n_1 n_2$$

(a) Lines are parallel, if  $l_1 / l_2 = m_1 / m_2 = n_1 / n_2$

(b) Lines are perpendicular, if  $l_1 l_2 + m_1 m_2 + n_1 n_2 = 0$

(v) If  $\theta$  is the angle between two lines whose direction ratios are proportional to  $a_1, b_1, c_1$  and  $a_2, b_2, c_2$  respectively, then the angle  $\theta$  between them is given by

$$\cos \theta = a_1 a_2 + b_1 b_2 + c_1 c_2 / \sqrt{a_1^2 + b_1^2 + c_1^2} \sqrt{a_2^2 + b_2^2 + c_2^2}$$

Lines are parallel, if  $a_1 / a_2 = b_1 / b_2 = c_1 / c_2$

Lines are perpendicular, if  $a_1 a_2 + b_1 b_2 + c_1 c_2 = 0$ .

(vi) The projection of the line segment joining points  $P(x_1, y_1, z_1)$  and  $Q(x_2, y_2, z_2)$  to the line having direction cosines  $l, m, n$  is

$$|(x_2 - x_1)l + (y_2 - y_1)m + (z_2 - z_1)n|.$$

(vii) The direction ratio of the line passing through points  $P(x_1, y_1, z_1)$  and  $Q(x_2, y_2, z_2)$  are proportional to  $x_2 - x_1, y_2 - y_1, z_2 - z_1$ . Then, direction cosines of PQ are

$$x_2 - x_1 / |PQ|, y_2 - y_1 / |PQ|, z_2 - z_1 / |PQ|$$

### Area of Triangle

If the vertices of a triangle be  $A(x_1, y_1, z_1)$  and  $B(x_2, y_2, z_2)$  and  $C(x_3, y_3, z_3)$ , then

$$\text{Area of } \triangle ABC = \frac{1}{2} \sqrt{\Delta x^2 + \Delta y^2 + \Delta z^2}$$

$$\text{where, } \Delta x = \frac{1}{2} \begin{vmatrix} y_1 & z_1 & 1 \\ y_2 & z_2 & 1 \\ y_3 & z_3 & 1 \end{vmatrix}, \Delta y = \frac{1}{2} \begin{vmatrix} x_1 & z_1 & 1 \\ x_2 & z_2 & 1 \\ x_3 & z_3 & 1 \end{vmatrix} \text{ and } \Delta z = \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

### Angle Between Two Intersecting Lines

If  $l(x_1, m_1, n_1)$  and  $l(x_2, m_2, n_2)$  be the direction cosines of two given lines, then the angle  $\theta$  between them is given by

$$\cos \theta = l_1 l_2 + m_1 m_2 + n_1 n_2$$

(i) The angle between any two diagonals of a cube is  $\cos^{-1} (1 / 3)$ .

(ii) The angle between a diagonal of a cube and the diagonal of a face (of the cube) is  $\cos^{-1} (\sqrt{2} / 3)$

### Straight Line in Space

The two equations of the line  $ax + by + cz + d = 0$  and  $a'x + b'y + c'z + d' = 0$  together represents a straight line.

1. Equation of a straight line passing through a fixed point  $A(x_1, y_1, z_1)$  and having direction ratios  $a, b, c$  is given by

$$x - x_1 / a = y - y_1 / b = z - z_1 / c, \text{ it is also called the symmetrically form of a line.}$$

Any point  $P$  on this line may be taken as  $(x_1 + \lambda a, y_1 + \lambda b, z_1 + \lambda c)$ , where  $\lambda \in \mathbb{R}$  is parameter. If  $a, b, c$  are replaced by direction cosines  $l, m, n$ , then  $\lambda$ , represents distance of the point  $P$  from the fixed point  $A$ .

2. Equation of a straight line joining two fixed points  $A(x_1, y_1, z_1)$  and  $B(x_2, y_2, z_2)$  is given by

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}$$

3. Vector equation of a line passing through a point with position vector  $\mathbf{a}$  and parallel to vector  $\mathbf{b}$  is  $\mathbf{r} = \mathbf{a} + \lambda \mathbf{b}$ , where  $\lambda$  is a parameter.

4. Vector equation of a line passing through two given points having position vectors  $\mathbf{a}$  and  $\mathbf{b}$  is  $\mathbf{r} = \mathbf{a} + \lambda (\mathbf{b} - \mathbf{a})$ , where  $\lambda$  is a parameter.

5. (a) The length of the perpendicular from a point  $P(\vec{\alpha})$  on the line  $\mathbf{r} = \mathbf{a} + \lambda \mathbf{b}$  is given by

$$\sqrt{|\vec{\alpha} - \mathbf{a}|^2 - \left\{ \frac{(\vec{\alpha} - \mathbf{a}) \cdot \mathbf{b}}{|\mathbf{b}|} \right\}^2}$$

(b) The length of the perpendicular from a point  $P(x_1, y_1, z_1)$  on the line

$$\frac{x - a}{l} = \frac{y - b}{m} = \frac{z - c}{n} \text{ is given by}$$

$$\sqrt{\frac{\{(a - x_1)^2 + (b - y_1)^2 + (c - z_1)^2\} - \{(a - x_1)l + (b - y_1)m + (c - z_1)n\}^2}{l^2 + m^2 + n^2}}$$

where,  $l, m, n$  are direction cosines of the line.

6. **Skew Lines** Two straight lines in space are said to be skew lines, if they are neither parallel nor intersecting.

7. **Shortest Distance** If  $l_1$  and  $l_2$  are two skew lines, then a line perpendicular to each of lines  $l_1$  and  $l_2$  is known as the line of shortest distance.

If the line of shortest distance intersects the lines  $l_1$  and  $l_2$  at  $P$  and  $Q$  respectively, then the distance  $PQ$  between points  $P$  and  $Q$  is known as the shortest distance between  $l_1$  and  $l_2$ .

8. The shortest distance between the lines

and

$$\frac{x - x_1}{l_1} = \frac{y - y_1}{m_1} = \frac{z - z_1}{n_1}$$

$$\frac{x - x_2}{l_2} = \frac{y - y_2}{m_2} = \frac{z - z_2}{n_2} \text{ is given by}$$

$$d = \frac{\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix}}{\sqrt{(m_1 n_2 - m_2 n_1)^2 + (n_1 l_2 - n_2 l_1)^2 + (l_1 m_2 - l_2 m_1)^2}}$$

9. The shortest distance between lines  $r = a_1 + \lambda b_1$  and  $r = a_2 + \mu b_2$  is given by

$$d = \frac{|(b_1 \times b_2) \cdot (a_2 - a_1)|}{|b_1 \times b_2|}$$

10. The shortest distance parallel lines  $r = a_1 + \lambda b_1$  and  $r = a_2 + \mu b_2$  is given by

$$d = \frac{|(a_2 - a_1) \times b|}{|b|}$$

11. Lines  $r = a_1 + \lambda b_1$  and  $r = a_2 + \mu b_2$  are intersecting lines, if  $(b_1 * b_2) * (a_2 - a_1) = 0$ .

12. The image or reflection  $(x, y, z)$  of a point  $(x_1, y_1, z_1)$  in a plane  $ax + by + cz + d = 0$  is given by

$$x - x_1 / a = y - y_1 / b = z - z_1 / c = -2(ax_1 + by_1 + cz_1 + d) / a^2 + b^2 + c^2$$

13. The foot  $(x, y, z)$  of a point  $(x_1, y_1, z_1)$  in a plane  $ax + by + cz + d = 0$  is given by

$$x - x_1 / a = y - y_1 / b = z - z_1 / c = -(ax_1 + by_1 + cz_1 + d) / a^2 + b^2 + c^2$$

14. Since, x, y and z-axes pass through the origin and have direction cosines  $(1, 0, 0)$ ,  $(0, 1, 0)$  and  $(0, 0, 1)$ , respectively. Therefore, their equations are

$$x - \text{axis} : x - 0 / 1 = y - 0 / 0 = z - 0 / 0$$

$$y - \text{axis} : x - 0 / 0 = y - 0 / 1 = z - 0 / 0$$

$$z - \text{axis} : x - 0 / 0 = y - 0 / 0 = z - 0 / 1$$

## Plane

A plane is a surface such that, if two points are taken on it, a straight line joining them lies wholly in the surface.

### General Equation of the Plane

The general equation of the first degree in x, y, z always represents a plane. Hence, the general equation of the plane is  $ax + by + cz + d = 0$ . The coefficient of x, y and z in the cartesian equation of a plane are the direction ratios of normal to the plane.

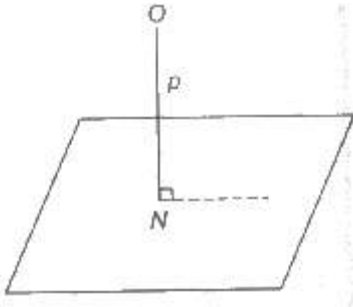
### Equation of the Plane Passing Through a Fixed Point

The equation of a plane passing through a given point  $(x_1, y_1, z_1)$  is given by  $a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$ .

## Normal Form of the Equation of Plane

(i) The equation of a plane, which is at a distance  $p$  from origin and the direction cosines of the normal from the origin to the plane are  $l, m, n$  is given by  $lx + my + nz = p$ .

(ii) The coordinates of foot of perpendicular  $N$  from the origin on the plane are  $(lp, mp, np)$ .



## Intercept Form

The intercept form of equation of plane represented in the form of

$$x/a + y/b + z/c = 1$$

where,  $a, b$  and  $c$  are intercepts on  $X, Y$  and  $Z$ -axes, respectively.

**For x intercept** Put  $y = 0, z = 0$  in the equation of the plane and obtain the value of  $x$ . Similarly, we can determine for other intercepts.

## Equation of Planes with Given Conditions

(i) Equation of a plane passing through the point  $A(x_1, y_1, z_1)$  and parallel to two given lines with direction ratios

$$a_1, b_1, c_1 \text{ and } a_2, b_2, c_2 \text{ is } \begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0.$$

(ii) Equation of a plane through two points  $A(x_1, y_1, z_1)$  and  $B(x_2, y_2, z_2)$  and parallel to a line with direction ratios  $a, b, c$  is

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a & b & c \end{vmatrix} = 0.$$

(iii) The Equation of a plane passing through three points  $A(x_1, y_1, z_1), B(x_2, y_2, z_2)$  and  $C(x_3, y_3, z_3)$  is

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix} = 0.$$

(iv) Four points  $A(x_1, y_1, z_1)$ ,  $B(x_2, y_2, z_2)$ ,  $C(x_3, y_3, z_3)$  and  $D(x_4, y_4, z_4)$  are coplanar if and only if

$$\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \\ x_4 - x_1 & y_4 - y_1 & z_4 - z_1 \end{vmatrix} = 0.$$

(v) Equation of the plane containing two coplanar lines

$$\begin{aligned} & \frac{x - x_1}{a_1} = \frac{y - y_1}{b_1} = \frac{z - z_1}{c_1} \\ \text{and} & \frac{x - x_2}{a_2} = \frac{y - y_2}{b_2} = \frac{z - z_2}{c_2} \text{ is} \\ & \begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0. \end{aligned}$$

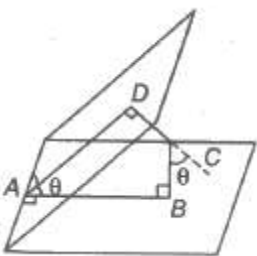
### Angle between Two Planes

The angle between two planes is defined as the angle between the normal to them from any point.

Thus, the angle between the two planes

$$a_1x + b_1y + c_1z + d_1 = 0$$

$$\text{and } a_2x + b_2y + c_2z + d_2 = 0$$



is equal to the angle between the normals with direction cosines

$$\pm a_1 / \sqrt{\sum a_1^2}, \pm b_1 / \sqrt{\sum a_1^2}, \pm c_1 / \sqrt{\sum a_1^2}$$

and  $\pm a_2 / \sqrt{\Sigma a_2^2}, \pm b_2 / \sqrt{\Sigma a_2^2}, \pm c_2 / \sqrt{\Sigma a_2^2}$

If  $\theta$  is the angle between the normals, then

$$\cos \theta = \pm a_1 a_2 + b_1 b_2 + c_1 c_2 / \sqrt{a_1^2 + b_1^2 + c_1^2} \sqrt{a_2^2 + b_2^2 + c_2^2}$$

### Parallelism and Perpendicularity of Two Planes

Two planes are parallel or perpendicular according as the normals to them are parallel or perpendicular.

Hence, the planes  $a_1 x + b_1 y + c_1 z + d_1 = 0$  and  $a_2 x + b_2 y + c_2 z + d_2 = 0$

are parallel, if  $a_1 / a_2 = b_1 / b_2 = c_1 / c_2$  and perpendicular, if  $a_1 a_2 + b_1 b_2 + c_1 c_2 = 0$ .

Note The equation of plane parallel to a given plane  $ax + by + cz + d = 0$  is given by  $ax + by + cz + k = 0$ , where  $k$  may be determined from given conditions.

### Angle between a Line and a Plane

**In Vector Form** The angle between a line  $r = a + \lambda b$  and plane  $r \cdot n = d$ , is defined as the complement of the angle between the line and normal to the plane:

$$\sin \theta = n \cdot b / |n||b|$$

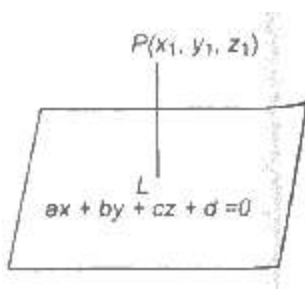
**In Cartesian Form** The angle between a line  $x - x_1 / a_1 = y - y_1 / b_1 = z - z_1 / c_1$

and plane  $a_2 x + b_2 y + c_2 z + d_2 = 0$  is  $\sin \theta = a_1 a_2 + b_1 b_2 + c_1 c_2 / \sqrt{a_1^2 + b_1^2 + c_1^2} \sqrt{a_2^2 + b_2^2 + c_2^2}$

### Distance of a Point from a Plane

Let the plane in the general form be  $ax + by + cz + d = 0$ . The distance of the point  $P(x_1, y_1, z_1)$  from the plane is equal to

$$\left| \frac{ax_1 + by_1 + cz_1 + d}{\sqrt{a^2 + b^2 + c^2}} \right|$$



If the plane is given in, normal form  $lx + my + nz = p$ . Then, the distance of the point  $P(x_1, y_1, z_1)$  from the plane is  $|lx_1 + my_1 + nz_1 - p|$ .

### Distance between Two Parallel Planes

If  $ax + by + cz + d_1 = 0$  and  $ax + by + cz + d_2 = 0$  be equation of two parallel planes. Then, the distance between them is

$$\left| \frac{d_2 - d_1}{\sqrt{a^2 + b^2 + c^2}} \right|$$

### Bisectors of Angles between Two Planes

The bisector planes of the angles between the planes

$a_1x + b_1y + c_1z + d_1 = 0$ ,  $a_2x + b_2y + c_2z + d_2 = 0$  is

$$a_1x + b_1y + c_1z + d_1 / \sqrt{\Sigma a_1^2} = \pm a_2x + b_2y + c_2z + d_2 / \sqrt{\Sigma a_2^2}$$

One of these planes will bisect the acute angle and the other obtuse angle between the given plane.

### Sphere

A sphere is the locus of a point which moves in a space in such a way that its distance from a fixed point always remains constant.

### General Equation of the Sphere

**In Cartesian Form** The equation of the sphere with centre  $(a, b, c)$  and radius  $r$  is

$$(x - a)^2 + (y - b)^2 + (z - c)^2 = r^2 \dots\dots(i)$$

In generally, we can write

$$x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$$

Here, its centre is  $(-u, -v, -w)$  and radius  $= \sqrt{u^2 + v^2 + w^2 - d}$

**In Vector Form** The vector equation of a sphere of radius  $a$  and Centre having position vector  $c$  is  $|r - c| = a$

### Important Points to be Remembered

(i) The general equation of second degree in  $x, y, z$  is  $ax^2 + by^2 + cz^2 + 2hxy + 2kyz + 2lzx + 2ux + 2vy + 2wz + d = 0$

represents a sphere, if

(a)  $a = b = c (\neq 0)$

(b)  $h = k = l = 0$

The equation becomes

$$ax^2 + ay^2 + az^2 + 2ux + 2vy + 2wz + d = 0 \dots(A)$$

To find its centre and radius first we make the coefficients of  $x^2$ ,  $y^2$  and  $z^2$  each unity by dividing throughout by  $a$ .

Thus, we have

$$x^2 + y^2 + z^2 + (2u/a)x + (2v/a)y + (2w/a)z + d/a = 0 \dots(B)$$

$$\therefore \text{Centre is } (-u/a, -v/a, -w/a)$$

$$\text{and radius} = \sqrt{u^2/a^2 + v^2/a^2 + w^2/a^2 - d/a}$$

$$= \sqrt{u^2 + v^2 + w^2 - ad} / |a| .$$

(ii) Any sphere concentric with the sphere

$$x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$$

$$\text{is } x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + k = 0$$

(iii) Since,  $r^2 = u^2 + v^2 + w^2 - d$ , therefore, the Eq. (B) represents a real sphere, if  $u^2 + v^2 + w^2 - d > 0$

(iv) The equation of a sphere on the line joining two points  $(x_1, y_1, z_1)$  and  $(x_2, y_2, z_2)$  as a diameter is

$$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) + (z - z_1)(z - z_2) = 0.$$

(v) The equation of a sphere passing through four non-coplanar points  $(x_1, y_1, z_1)$ ,  $(x_2, y_2, z_2)$ ,  $(x_3, y_3, z_3)$  and  $(x_4, y_4, z_4)$  is

$$\begin{vmatrix} x^2 + y^2 + z^2 & x & y & z & 1 \\ x_1^2 + y_1^2 + z_1^2 & x_1 & y_1 & z_1 & 1 \\ x_2^2 + y_2^2 + z_2^2 & x_2 & y_2 & z_2 & 1 \\ x_3^2 + y_3^2 + z_3^2 & x_3 & y_3 & z_3 & 1 \\ x_4^2 + y_4^2 + z_4^2 & x_4 & y_4 & z_4 & 1 \end{vmatrix} = 0$$

### Tangency of a Plane to a Sphere

The plane  $lx + my + nz = p$  will touch the sphere  $x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$ , if length of the perpendicular from the centre  $(-u, -v, -w) =$  radius,

$$\text{i.e., } |lu - mv - nw - p| / \sqrt{l^2 + m^2 + n^2} = \sqrt{u^2 + v^2 + w^2 - d}$$

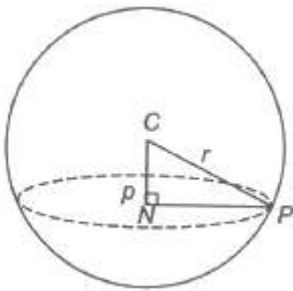
$$(lu - mv - nw - p)^2 = (u^2 + v^2 + w^2 - d) (l^2 + m^2 + n^2)$$

### Plane Section of a Sphere

Consider a sphere intersected by a plane. The set of points common to both sphere and plane is called a plane section of a sphere.

$$\text{In } \triangle CNP, NP^2 = CP^2 - CN^2 = r^2 - p^2$$

$$\therefore NP = \sqrt{r^2 - p^2}$$



Hence, the locus of P is a circle whose centre is at the point N, the foot of the perpendicular from the centre of the sphere to the plane.

The section of sphere by a plane through its centre is called a great circle. The centre and radius of a great circle are the same as those of the sphere.